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CONDITIONS FOR DEVELOPMENT OF CAVITATION IN SCLERONOMOUS MEDIA

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An energy inequality that determines the conditions for development of cavitation in scleronomous media in the range from liquid (bitumens, paints, and gels) to solid (lead, aluminum, copper, etc.) plastic media is constructed upon pulse tension in terms of the viscoelastic-plastic model. A relation that allows one to determine the time of negative-pressure relaxation during the growth of cavitating pores in a medium is derived. With allowance for the previously obtained conditions for development of bubble cavitation in a relaxing field of negative pressure in shock wave-loaded liquids, this result allows one to separate a class of condensed media capable to cavitate under pulse loading.

It is known that the dynamics of fracture of condensed media is strongly affected by the development of bubble cavitation in them at the stage of unloading. In particular, the growth of bubbles (or pores) results in the relaxation of stretching stresses. Even under intense shock-wave loading, this relaxation blocks brittle fracture of the sample: its fragmentation occurs in a viscous-fracture regime. It has been found experimentally [1, 2] that upon pulse volume tension of the sample (at the stage of unloading after shock-wave loading), unbounded cavitation can develop both in low-viscosity liquids and in high-viscosity emulsions, suspensions (pastes), and gels having a liquid low-viscosity matrix. At the same time, it has been shown that in the samples of high-viscosity Newtonian liquids, the development of apparent cavitation is not observed under similar loading conditions. According to [3, 4], the dynamic fracture of plastic metals is accompanied by the growth of almost spherical pores. In this connection, the determination of conditions satisfied by the loading parameters at which a cavitational process of a given intensity is developed in the media under study is of interest.

The conditions for development of bubble cavitation in liquid media of arbitrary viscosity upon pulse volume tension owing to shock-wave loading were determined in [5]. In the present work, the conditions under which cavitation develops in the case of pulse loading of the samples of scleronomous viscoelastic-plastic media are determined. These media behave as Hooke's elastic body at a shear stress τ smaller than the threshold value τ_* corresponding to the onset of plastic flow and go to a viscoplastic state for $\tau > \tau_*$. (Among these media are plastic metals, bitumens, asphalts, paints, gels, etc.) We consider that the scleronomous medium is a cavitating medium at the specified parameters of shock-wave loading if the volume concentration of cavitational hollows α increases by at least an order of magnitude compared to the initial value α_0 at the stage of its unloading in the stretching-stress zone.

1. Since all the condensed media possess volume elasticity, the rheological model of their volume tension can be presented as follows. If a medium had an ideally homogeneous structure (Fig. 1a), its reaction to the volume strain ε_V (tension) could be described by the mechanical model shown in Fig. 1b. In this case, the thermodynamic equilibrium in the medium is disturbed upon rapid volume tension and it is reached again for a certain time T_0. Here, in the process of equilibrium re-establishment, the pressure that opposes the

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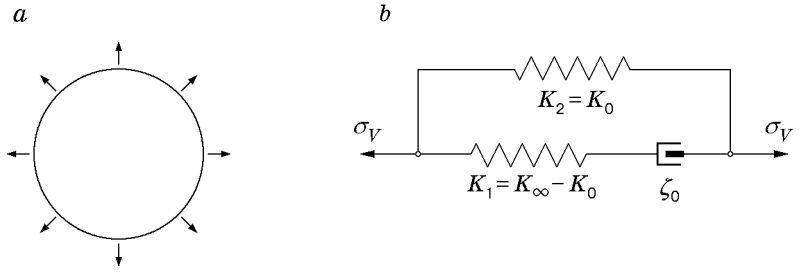

Fig. 1

change in the volume of the medium, decreases, and, hence, the modulus of volume elasticity decreases from the maximum dynamic value K_∞ to the static value K_0. The thermodynamic equilibrium is re-established owing to restructuring of the medium at the molecular level, which is accompanied by partial dissipation of the elastic energy of the medium owing to its volume viscosity ζ_0 (unloading of the spring K_1 on the piston ζ_0 in the mechanical model). Therefore, in this case, the time of tensile-stress relaxation from the initial value $\sigma_V(0)$ to the equilibrium value σ_V^0 can be presented in the form of the dependence $T_0 = \zeta_0/(K_\infty - K_0)$. If σ_V^0 is lower than the threshold value of the volume yield of the medium σ_V^* (determined by intermolecular or interatomic bonds in this case), and the maximum shear stress is $(\sigma_i - \sigma_j)/2 = \tau_m < \tau_*$ (σ_i and σ_j are the principal values of the tensile-stress tensor and τ_* is the yield point), the stress $\sigma_V^0 = K_0\varepsilon_V$ will occur for an infinitely long period in an ideally homogeneous medium at the stage of $t > T_0$ for $\varepsilon_V = \text{const}$.

Real condensed media (Fig. 2a) always have structural defects: liquids contain cavitational nuclei, and scleronomous media contain dislocations and micropores. As a result, the elastic stresses relax in such media almost at any values of τ_m. In liquid media, we have $\tau_*/P_\infty \ll 1$, where P_∞ is the hydrostatic pressure, and the growth of a cavitational nucleus, i.e., the divergent flow of the liquid which determines tensile-stress relaxation, begins when the pressure difference in the bubble and the liquid matrix is positive (with allowance for surface tension). In scleronomous media, for $\tau_m < \tau_*$, the motion of vacancies has a diffuse character, and relaxation occurs in a creep regime; for $\tau_m > \tau_*$, the above-barrier gliding of vacancies, which determines the plastic-deformation regime of the medium and, hence, stress relaxation, occurs.

With allowance for the aforesaid, we formulate the problem of determining the conditions under which cavitation develops in scleronomous media and the time of tensile-stress relaxation. We assume that the sample of the material to be examined having, for definiteness, a spherical or cylindrical symmetry, contains monodisperse micropores of radius R_0 uniformly distributed over the sample. The initial volume concentration of the micropores is α_0, and their counting concentration is $n = 3\alpha_0/(4\pi R_0^3)$. We separate a cubic ($1\times1\times1$ cm) unit volume in the sample and divide it into n identical cubic cells with a micropore at the center of each cell (Fig. 2a). The cell edge is $l = n^{-1/3} = R_0[4\pi(3\alpha_0)^{-1}]^{1/3}$ long; by virtue of the smallness of α_0, we have $L \gg l \gg R_0$, where L is the characteristic dimension of the sample. The physical processes in the medium can occur on three space scales. Tensile-stress relaxation, which is the process of reaching thermodynamic equilibrium in a macroscopic physical system, occurs on the L scale, the pore sizes change on the l scale, and the plastic deformation of the material in the neighborhood of each pore occurs on the scale of interatomic or intermolecular interactions owing to the motion of dislocations.

Figure 2b shows schematically the mechanical model that corresponds to macroscopic processes on the L scale. The sample to be examined is assumed to be loaded by a one-dimensional, coaxial, and divergent from its center of symmetry shock wave (SW) of the type $P(\hat{r}, t) = P_*(\hat{r}, 0)\exp(-t/\lambda(\hat{r}))$, where $\hat{r}$ is the distance from the center of symmetry of the sample, $P_*(\hat{r}, 0)$ is the pressure in the SW front at the moment of its motion from the center of symmetry at the distance $\hat{r}$, and $\lambda(\hat{r})$ is the constant of the time of pressure drop behind the SW front for a fixed $\hat{r}$. After the SW front reaches the free surface of the sample, at the unloading stage the free surface displaces, and a rarefaction wave which converges to the center of the sample propagates into the sample; as a result, the volume strain of the sample ε_V occurs (in the model, the springs K_1, K_2, and K_3 are stretched) and a tensile-stress field $\sigma_V = (K_1 + K_2 + K_3)\varepsilon_V = K_\infty\varepsilon_V$ forms. With ε_V

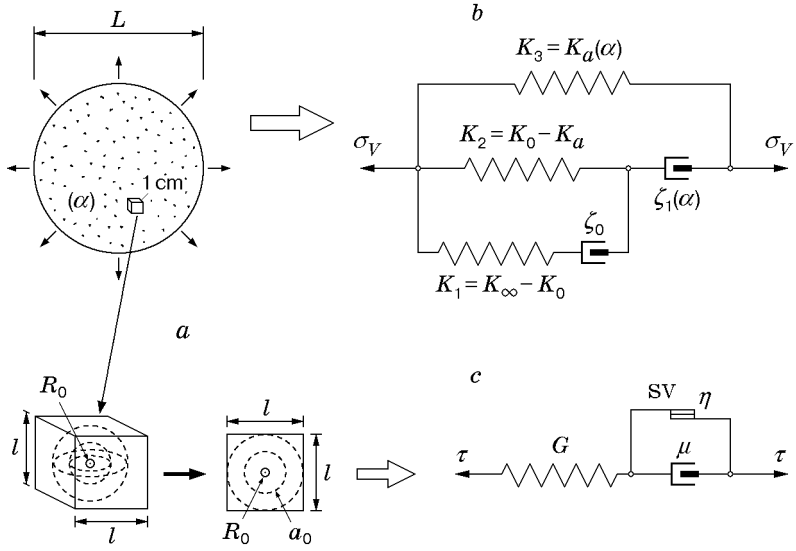

Fig. 2

remained constant, the stresses relax partially owing to the transition of the medium in a thermodynamic-equilibrium state for $T_0 = \zeta_0(K_\infty - K_0)^{-1}$, and the stress decreases to $\sigma_V = (K_2 + K_3)\varepsilon_V = K_0\varepsilon_V$ [usually, $(K_\infty - K_0)K_0^{-1} \ll 1$ for scleronomous media]. Then, since the shear stresses $\tau = \sigma_i - \sigma_j$ appear on the l scale in the vicinity of the pore for $\tau_m > \tau_*$, the medium goes to a viscoplastic state and pore extension begins. On the mechanical model of this process (Fig. 2c), the Saint-Venant (SV) element is opened, and the spring G, which corresponds to the shear elasticity of the medium, is unloaded on the piston μ which characterizes its plastic viscosity; in the general cases, the plastic viscosity depends on the strain rate, the concentration of vacancies, the temperature, and other physical parameters of the process.

We formulate a rheological equation of state that corresponds to the shear deformation of a medium in the neighborhood of a pore. Based on the mechanical model of a medium (Fig. 2c), which represents a sequence of the mechanical models of Hooke's and Binham's bodies, one can write the total value of the strain deviator e_{ij} in the form

$$e_{ij} = \frac{\tilde{\sigma}_{ij}}{2G} + \tilde{\sigma}_{ij}\Big[2(\mu + \eta)\frac{\partial}{\partial t}\Big]^{-1},$$

whence

$$\dot{\tilde{\sigma}}_{ij} + \frac{G}{\mu + \eta}\,\tilde{\sigma}_{ij} = 2G\dot{e}_{ij}. \tag{1}$$

Here $\tilde{\sigma}_{ij}$ is the strain deviator, η is the effective-viscosity coefficient of the medium upon passage of the intensity of the tangential stress $\tau_i = (\tilde{\sigma}_{ij}\tilde{\sigma}_{ij}/2)^{1/2}$ through the value of the limiting shear stress τ_*, and μ is the shear-viscosity coefficient of the medium for $\tau_i > \tau_*$, i.e., in a plastic-flow regime. Because when the plastic element η of the complex mechanical model reaches the limiting shear stress [6], the viscous element parallel to it μ does not render resistance to deformation, we assume that $\mu = 0$ in (1); multiplying this equation by $\tilde{\sigma}_{ij}$, we obtain $\eta = \tau_*(2\dot{e}_{ij}\dot{e}_{ij} - \dot{e}_{ij}\dot{\tilde{\sigma}}_{ij}G^{-1})^{-1/2}$ with allowance for $\tilde{\sigma}_{ij}\dot{\tilde{\sigma}}_{ij} = 0$ and the Mises condition $\tilde{\sigma}_{ij}\tilde{\sigma}_{ij} = 2\tau_*^2$ or $\eta = \tau_*\{\dot{e}_i[1 - \dot{\tilde{\sigma}}_{ij}/(2G\dot{e}_{ij})]^{1/2}\}^{-1}$ with allowance for the expressions for the generalized shear-strain rate $\dot{e}_i = (2\dot{e}_{ij}\dot{e}_{ij})^{1/2}$. Substituting the latter expression into (1), we obtain the general form of the rheological equation for shear strain of the medium

$$\tilde{\sigma}_{ij} + \Big[\mu + \tau_*\dot{e}_i^{-1}\Big(1 - \frac{\dot{\tilde{\sigma}}_{ij}}{2G\dot{e}_{ij}}\Big)^{-1/2}\Big]\frac{\dot{\tilde{\sigma}}_{ij}}{G} = 2\tau_*\dot{e}_{ij}\dot{e}_i^{-1}\Big(1 - \frac{\dot{\tilde{\sigma}}_{ij}}{2G\dot{e}_{ij}}\Big)^{-1/2} + 2\mu\dot{e}_{ij} \tag{2}$$

for $\tau_i > \tau_*$ or

$$\tau + \left[\mu + \tau_*\Big(\dot{\varepsilon}_\tau^2 - \frac{\dot{\tau}\dot{\varepsilon}_\tau}{2G}\Big)^{-1/2}\frac{\dot{\varepsilon}_\tau}{|\dot{\varepsilon}_\tau|}\right]\frac{\dot{\tau}}{G} = \tau_*\Big(1 - \frac{\dot{\tau}}{2G\dot{\varepsilon}_\tau}\Big)^{-1/2}\frac{\dot{\varepsilon}_\tau}{|\dot{\varepsilon}_\tau|} + \mu\dot{\varepsilon}_\tau \tag{3}$$

in the case of pure shear ($\dot{\varepsilon}_\tau$ is the pure-shear strain rate).

For $\tau < \tau_*$, the SV element is closed; therefore, $\eta = \infty$ and the equations of state (2) and (3) take the form $\tilde{\sigma}_{ij} = 2Ge_{ij}$ and $\tau = G\varepsilon_\tau$, which corresponds to the case of Hooke's elastic body.

For $\tau > \tau_*$, the SV element is opened; therefore, on the mechanical model, the further expansion of the spring does not occur: strain occurs only owing to the displacement of the piston μ, which corresponds to a viscoplastic flow of the medium. Therefore, for $\tau > \tau_*$, the element G is not subjected to deformation on the mechanical model; assuming that $G = \infty$ in (2) and (3), we obtain the following equation of state of a viscoplastic medium in the vicinity of pores: $\tilde{\sigma}_{ij} = 2\tau_*\dot{e}_{ij}/\dot{e}_i + 2\mu\dot{e}_{ij}$ or

$$\tau = \tau_* \frac{\dot{\varepsilon}_\tau}{|\dot{\varepsilon}_\tau|} + \mu\dot{\varepsilon}_\tau = \tau_* \operatorname{sign}\dot{\varepsilon}_\tau + 2\mu\dot{\varepsilon}_\tau. \tag{4}$$

Since upon expansion of the medium, the stress tensor σ_{ij} has a diagonal form in any coordinate system, for the spherical coordinate system (r, θ, φ), whose coordinate origin is aligned with the center of a pore, Eq. (4), which is the constitutive equation of a medium in a viscoplastic-flow regime in the vicinity of a spherical pore, takes the following form:

$$\sigma_{rr} - \sigma_{\theta\theta} = -\tau_* + 2\mu\Big(\frac{\partial u}{\partial r} - \frac{u}{r}\Big). \tag{5}$$

Here u is the radial plastic-flow rate of the medium.

2. Based on the aforesaid, we consider the problem of growth conditions for a pore contained in the ith cell on the l scale in the following formulation. The center of the pore coincides with that of the cell; the distance from the cell's center to the center of symmetry of the sample is $\hat{r}_i$. We assume that P_1 is the gas pressure in a micropore of radius R_0 and that the negative pressure $-\tilde{P}_i^0$ is applied to the cell material (the time for which the pressure $-\tilde{P}_i^0$ is reached on the entire scale of the cell is ignored). In addition, the hydrostatic pressure $P_\infty > 0$ is applied to the entire volume of the sample, and the capillary pressure $P_L = 2\gamma R^{-1}$, where γ is the interphase-tension coefficient and R is the current radius of the pore, acts on the pore surface. Since the goal of this work is to determine the conditions for pore growth, our analysis omits the range of dynamic loading in which the pressure modulus $|-\tilde{P}_i^0|$ is so large at the stage of unloading that, undoubtedly, the entire medium in the cell goes to a viscoplastic state and the pore grows in an almost bubble-growth regime in a high-viscosity liquid. Therefore, we consider the cases of ultimately small values of $|-\tilde{P}_i^0|$ at which a viscoplastic flow of the medium and the pore growth are still possible.

With allowance for $R_0 \ll l$, using the solution of the stress-distribution problem in an elastic medium with a spherical cavity [7], for the boundary conditions

$$\sigma_{rr}\Big|_{r\to\infty} = -(-\tilde{P}_i^0 + P_\infty), \qquad \sigma_{rr}\Big|_{r=R} = -(-P_1 + 2\gamma/R), \tag{6}$$

we obtain

$$\sigma_{rr} = P_{i0} + P_i'/\bar{r}^3, \qquad \sigma_{\theta\theta} = P_{i0} - P_i'/(2\bar{r}^3), \tag{7}$$

where $P_{i0} = \tilde{P}_i^0 - P_\infty$, $P_i' = -\tilde{P}_i^0 - P_1 + P_\infty + 2\gamma R^{-1}$, and $\bar{r} = rR^{-1}$.

According to the Tresca condition [8], a plastic flow will occur in the medium if

$$\tau_m = 0.5(\sigma_{\theta\theta} - \sigma_{rr}) \geqslant \tau_*. \tag{8}$$

After substitution of (7) into (8), we obtain the following plastic-flow condition in the vicinity of the pore:

$$\tau_m = \frac{3}{4\bar{r}^3}\Big(\tilde{P}_i^0 + P_1 - P_\infty - \frac{2\gamma}{R}\Big) \geqslant \tau_*. \tag{9}$$

If (9) is also satisfied for $\bar{r} > 1$, the outer radius $r = a$ of the spherical viscoplastic layer (see Fig. 2a), in which the pore extension is possible, is determined from (9) by the expression

$$a = R(3\hat{P}_i/(4\tau_*))^{1/3}, \tag{10}$$

where $\hat{P}_i = -P_i' = \tilde{P}_i^0 + P_1 - P_\infty - P_L$. It follows from (9) and (10) that, if, for $\hat{P}_i = \text{const}$, at $R = R_0$ the plastic-flow condition is satisfied in the layer $R_0 \leqslant r \leqslant a_0$, then we have $\tau_m(aR^{-1}) \geqslant \tau_*$ for any

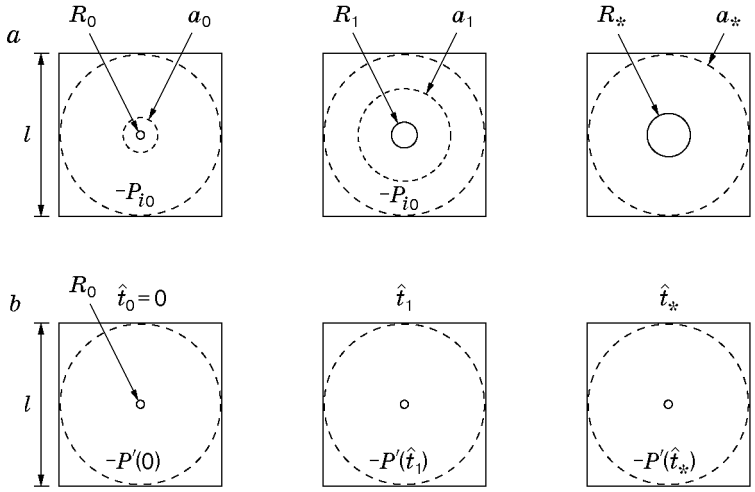

Fig. 3

$R > R_0$, since $\hat{P}_i = \text{const}$ for $aR^{-1} = \text{const}$ as well, i.e., a increases with R. Generally, if $\hat{P}_i = \psi(\hat{t})$, then $aR^{-1} = [3(4\tau_*)^{-1}]^{1/3}\hat{P}^{1/3}(\hat{t})$; therefore, in the case where the function $\psi(\hat{t}) > 0$ decreases as $\hat{P}_i \to 4\tau_*/3$, the radius $a \to R$, i.e., the thickness of the plasticity zone tends to zero and the pore is "frozen" into the elastic medium. Here $\hat{t}$ is the time reckoned from the moment of arrival of the rarefaction-wave front at the ith cell. We note that in an almost incompressible viscoplastic medium, the pores can grow owing to the presence of structural defects in the medium.

The equation of motion of a viscoplastic medium in the layer $R \leqslant r \leqslant a$ with the average density ρ has the form

$$\rho\Big(\frac{\partial u}{\partial t} + u\,\frac{\partial u}{\partial r}\Big) = \frac{\partial}{\partial r}\,\sigma_{rr} + \frac{2(\sigma_{rr} - \sigma_{\theta\theta})}{r}. \tag{11}$$

By virtue of the spherical symmetry, the motion of the medium will be vortex-free; consequently, with allowance for its incompressibility, from the continuity equation we obtain the relation for the velocity field in the layer $R \leqslant r \leqslant a$:

$$u = \dot{R}R^2/r^2. \tag{12}$$

Substituting (5) and (12) into Eq. (11) and integrating it over the elastoplastic layer from R to a, with allowance for the boundary conditions (6), the first of which takes the form $\sigma_{rr}(r) = \tilde{P}_i^0 - P_\infty$ at the layer boundary $r = a$, we obtain the equation of pore growth in the layer $R \leqslant r \leqslant a$

$$(1-\beta_0)R\ddot{R} + \Big[\frac{3}{2} - 2\Big(1 - \frac{\beta_0^3}{4}\Big)\beta_0\Big]\dot{R}^2 + \frac{4\mu}{\rho}\,(1-\beta_0^3)\,\frac{\dot{R}}{R} = \frac{\hat{P}_i}{\rho}\Big(1 - \frac{2\tau_*}{\hat{P}_i}\,\ln\,\beta_0^{-1}\Big), \tag{13}$$

in which $\beta_0 = R_0 a_0^{-1} = Ra^{-1}$ is determined for $\hat{P}_i = \text{const}$ from (10). According to (13), for the pore to expand up to a certain given value of the radius R_1, the negative-pressure field in the cell should be subject to the condition

$$\hat{P}_i(\hat{t}) \geqslant 2\tau_*\,\ln\,\beta_0^{-1} \tag{14}$$

at least for $0 \leqslant \hat{t} < \hat{t}_1^0$ ($\hat{t}_1^0$ is the time for which the cell expands to a given radius). It is easy to verify that condition (14) is always satisfied if the inequality (9) holds.

Therefore, the processes that occur under the action of a negative pressure in a viscoelastic-plastic medium containing micropores can be presented on the l and L scales as follows. At the initial moment $\hat{t} = 0$, a field of elastic tensile stresses $-P_{i0}$ forms under the action of the negative pressure in the ith cell (Fig. 3a). Here the free specific energy of the medium can be determined from the known relation $f = 0.5\sigma_{ik}e_{ik}$; with allowance for the isotropic character of the medium and Hooke's generalized law $e_{ik} = [(1+\nu)\sigma_{ik} - \nu\sigma_{ll}\delta_{ik}]E^{-1}$, this relation is reduced to the form

$$f = 0.5(1+\nu)[\sigma_{ik}^2 - \nu\sigma_{ll}^2(1+\nu)^{-1}]E^{-1}.$$

Here E and ν are Young's modulus and Poisson's ratio, respectively. With allowance for $\sigma_{ik}^2 = I_1^2 - 2I_2$ and $\sigma_{ll}^2 = I_1^2$, where I_1 and I_2 is the first and second invariants of the tensor σ_{ik}, and in the case of volume expansion of the medium $I_2 = 0$, we rewrite the latter equality in the following form with the use of (7):

$$f = 0.5(\sigma_{rr}^2 + 2\sigma_{\theta\theta}^2)E^{-1} = 1.5P_{i0}^2[1 + 0.5(1+\xi)^2\bar{r}^{-6}]E^{-1}. \tag{15}$$

Here $\xi = (P_1 - P_L)(\tilde{P}_i^0 - P_\infty)^{-1}$.

We note that in the case of solid-plastic materials, we have $\tau_* \gg P_L$, and, consequently, $\tilde{P}_i^0 \gg P_L$ according to (14), i.e., $\xi \ll 1$. For liquid-plastic media, condition (14) is satisfied, beginning with values of $\tilde{P}_i^0$ of the order P_∞, i.e., the cases where ξ is about unity are possible in these media.

At the initial moment, the free elastic energy of the cell F_0 is concentrated in the spherical layer $R_0 \leqslant r \leqslant 0.5l$, where l is the diameter of the sphere inscribed into the cubic cell (see Figs. 2a and 3a). Therefore, integrating (15) over the volume of this layer within the limits $1 \leqslant \bar{r} \leqslant 0.5lR_0^{-1} = (2R_0\sqrt[3]{n})^{-1} = k$, we obtain the expression

$$F_0(\hat{t}) = 4\pi R_0^3 \int_1^k f(\bar{r})\bar{r}^2\, d\bar{r} = \pi(1 - 8nR_0^3)[(1+\xi)^2 + (4\pi nR_0^3)^{-1}]E^{-1}R_0^3P_{i0}^2 \quad (\hat{t} = 0);$$

after the replacement $E = 9GK_0(3K_0 + G)^{-1}$, where K_0 is the modulus of volume elasticity of the matrix, this expression is reduced to the form

$$F_0(\hat{t}) = \frac{\pi R_0^3}{3}(1 - 8nR_0^3)\Big[(1+\xi)^2 + \frac{1}{4nR_0^3}\Big]\Big(\frac{1}{3K_0} + \frac{1}{G}\Big)P_{i0}^2 \quad (\hat{t} = 0). \tag{16}$$

Then, the pore growth begins in the plastic-flow zone $R_0 < r \leqslant a_0$ formed in its neighborhood (Fig. 3a), where a_0 is determined from (10). Here the plastic zone borders on the region $a_0 < r < 0.5l$, in which the medium continues to be in an elastodeformed state. In the process of pore growth, the plastic-flow zone extends and, hence, the thickness of the "undisturbed" elastodeformed layer, in which the negative pressure preserves the constant value $-P'$, decreases. At last, at a certain moment of time $\hat{t}_*$, the thickness of this layer decreases to zero: the boundary of the plastic-flow zone goes beyond the boundary of the cell, i.e., the quantity a reaches the value $a_* = 0.5l = 0.5n^{-1/3}$. From this moment, the constant-pressure condition is no longer satisfied at the boundary of a_*. By analogy with (16), we construct a relation that determines the value of the free elastic energy at any moment $0 < \hat{t}_1 < \hat{t}_*$ in the layer $a_1 < r \leqslant a_*$. Integrating $f(\bar{r})$ over $\bar{r}$ in the range $a_1R_0^{-1} = b_1\beta_0^{-1} < \bar{r} < a_*R_0^{-1} = (2R_0n^{1/3})^{-1}$, we obtain

$$F(\hat{t}_1) = \frac{\pi R_0^3}{3}\Big[(1+\xi)^2\Big(\frac{\beta_0^3}{b_1^3} - 8nR_0^3\Big) + \frac{1 - 8nR_0^3b_1^3\beta_0^{-3}}{4nR_0^3}\Big]\Big(\frac{1}{3K_0} + \frac{1}{G}\Big)P_{i0}^2. \tag{17}$$

Here, with allowance for $\beta_0^3 \ll 1$ and $\hat{P}_i(\hat{t}) = \tilde{P}_i^0[U(\hat{t}) - U(\hat{t} - \hat{t}_*)] + P_1 - P_\infty - P_L$ (U is a unit function), the quantity b_1 is determined on the segment $0 \leqslant \hat{t} \leqslant \hat{t}_*$ from the solution of Eq. (13) written in the form

$$b\ddot{b} + \frac{3 - 4\beta_0}{2(1-\beta_0)}\dot{b}^2 + h\,\frac{\dot{b}}{b} = Qh(1 - S), \tag{18}$$

where $h = 4\mu/[\rho R_0^2(1-\beta_0)]$, $Q = \hat{P}_i/(4\mu)$, $D = 2\tau_*/\hat{P}_i$, and $S = D\ln\beta_0^{-1}$. A qualitative analysis shows that, on the time interval $(0, \hat{t}_*)$, the inertial term in this equation (i.e., the term containing $\dot{b}^2$) is several orders of magnitude smaller than the other terms of the equation, and that monotonically decreasing, the acceleration $\ddot{b}$ has the same order of magnitude as the inertial term at a certain moment of time $\hat{t}_0$. With allowance for this, assuming that $b \simeq 1$ and $\hat{P}_i \simeq \tilde{P}_i^0 - P_\infty$, we reduce (18) to the following form at the initial stage of pore growth $0 < \hat{t} \leqslant \hat{t}_0$:

$$\ddot{b} + h\dot{b} = Q(1 - S)h. \tag{19}$$

Ignoring the term $\ddot{b}$, at the stage of $\hat{t} \geqslant \hat{t}_0$, we have

$$\dot{b} = Q(1-S)b. \tag{20}$$

Solving (19) and (20) on the time intervals $0 \leqslant \hat{t} \leqslant \hat{t}_0$ and $\hat{t} > \hat{t}_0$, respectively, we obtain

$$b(t) = \begin{cases} 1 + M(t + \mathrm{e}^{-t} - 1), & 0 \leqslant t \leqslant t_0, \\ b(t_0)\exp\left[M(t - t_0)\right], & t > t_0; \end{cases} \tag{21}$$

$$\dot{b}(t) = \begin{cases} Q(1-S)(1-\mathrm{e}^{-t}), & 0 \leqslant t \leqslant t_0, \\ Q(1-S)b(t), & t > t_0, \end{cases} \tag{22}$$

where $t = h\hat{t}$ and $M = Q(1-S)h^{-1}$. Since $\hat{t}_0$ is the moment of time at which, decreasing, $\ddot{b}$ is comparable, in order of magnitude, with $|3-4\beta_0|\dot{b}^2[2(1-\beta_0)b]^{-1}$, one can assume that $\hat{t}_0$ is the moment of time at which the equality $b(\hat{t}_0)\ddot{b}(\hat{t}_0) = 0.5(3-4\beta_0)(1-\beta_0)^{-1}\dot{b}^2(\hat{t}_0)$ holds. Substituting the values of b, $\dot{b}$, and $\ddot{b}$ from (21) and (22) into this equation, with allowance for $\hat{t}_0 = h^{-1}t_0$, we obtain the relation $(3-4\beta_0)\mathrm{e}^{t_0} + (1-2\beta_0)\mathrm{e}^{-t_0} - 2(1-\beta_0)t_0 = 2(1-\beta_0)M^{-1} + 2(2-3\beta_0)$ from which t_0 is determined.

Thus, during the pore growth in the cell, at least prior to the moment $\hat{t}_*$ the elastic layer $a < r \leqslant 0.5l$ (Fig. 3a) whose inner boundary moves according to the law (10) and the outer boundary remains immovable, i.e., the cell size and the average density of the medium remain constant, is preserved at the periphery of the viscoplastic zone $R \leqslant r \leqslant a$. Since the volume and density of the sample remain constant and the free energy decreases at this stage of pore growth, the process should be accompanied by a decrease in the volume-average negative pressure $-p$. To estimate the time of its relaxation T_1, we introduce the concept of an energetically equivalent medium (EM) on the time interval $0 \leqslant \hat{t} \leqslant \hat{t}_*$, the sample of which has the same size L and density and is divided into cells of the same size (Fig. 3b) containing pores of the same radius R_0 as in the case of the sample under study (Fig. 3a). At the initial moment of time, the store of free elastic energy in the EM cell also equals F_0, and $F(\hat{t})$ decreases in the cell with time according to the same law as in the cell of the sample. However, in contrast to the latter, this process in the EM cell occurs at a constant value of the micropore radius R_0, i.e., the free elastic energy is spent for internal restructuring of the EM whose thermodynamic equilibrium was disturbed by volume tension rather than for pore growth. Here the EM elastic energy decreases and, hence, the negative pressure in the cell, which coincides in this medium with the average nonvolume pressure $-p$, also decreases.

It is evident that $F(\hat{t})$ in the EM cell is determined from a formula similar to (17) [but containing $-P_{i0} = \text{const}$ instead of $-p = -P_{i0}\exp(-\hat{t}T_1^{-1})$]:

$$F(\hat{t}) = \frac{\pi R_0^3}{3}(1 - 8nR_0^3)\Big[(1+\xi)^2 + \frac{1}{4aR_0^3}\Big]\Big(\frac{1}{3K_0} + \frac{1}{G}\Big)P_{i0}^2\exp\Big(-\frac{2\hat{t}}{T_1}\Big). \tag{23}$$

Since at any moment of time $0 \leqslant \hat{t}_0 \leqslant \hat{t}_*$, the value of $F(\hat{t})$ in the cells of the medium considered and the EM should be the same (by definition of the EM), equating (17) and (23) at $\hat{t} = T_1 < \hat{t}_*$, we have

$$b(T_1) = \left[\frac{B\beta_0^3}{4}\left(1 + \sqrt{1 + \frac{8(1+\xi)^2}{B^2}}\right)\right]^{1/3},$$

$$B = \Big(1 - \frac{1}{e^2}\Big)\Big[\frac{1}{4nR_0^3} - 8(1+\xi)^2 nR_0^3\Big] - \left[(1+\xi)^2 - 1\right]\frac{1}{e^2}, \qquad e \approx 2.71828.$$

Equating this expression to the value of $b(T_1)$ from (21), we obtain a relation that determines the dependence of the relaxation time of the pressure $-p$ in an viscoelastic-plastic medium with pores:

$$T_1 = \hat{t}_0 + \frac{1}{3Q(1-S)}\ln\left[\frac{B\beta_0^3}{4}\left(1 + \sqrt{1 + \frac{8(1+\xi)^2}{B^2}}\right)\right].$$

By definition of $\hat{t}_0$, on the time interval $0 \leqslant \hat{t} \leqslant \hat{t}_0$, we have $b(\hat{t}) \simeq 1$ and, hence, $F(\hat{t}_0) \simeq F_0$, i.e., $p(\hat{t}_0) \simeq P_{i0}$; therefore, the latter equality is reduced to the form

$$T_1 \simeq \frac{1}{3Q(1-S)} \ln\left[\frac{B\beta_0^3}{4}\left(1+\sqrt{1+\frac{8(1+\xi)^2}{B^2}}\right)\right]. \tag{24}$$

The resulting dependence (24) allows one to determine the relaxation time of the cell-averaged value of the negative pressure $-p$; we note that relaxation occurs, because the free elastic energy F is spent for pore growth. However, the pore radius can grow to a certain value of $R_* = b_* R_0^{-1}$ provided the store of F in the layer $R_0 \leqslant r < a_* = \beta_0^{-1} b_*$ is not sufficient for execution of the work $\Delta W(b_*)$ to expand the pore in an viscoelastic-plastic medium to a given size. We determine the value of $\Delta W(b_*)$ by integrating (18) over time from $\hat{t}_0 = 0$ to the moment t_* corresponding to the value of $b_*(\hat{t}_*)$:

$$\frac{4\pi R_0^3}{3}(b_*^3-1)\hat{P}_i = \frac{4\pi R_0^3}{3}(b_*^3-1)2\tau_* \ln\beta_0^{-1} + 2\pi\rho R_0^5[(1-\beta_0)b_*^3\dot{b}_*^2 - \beta_0 J_1] + 16\pi R_0^3\mu J_2. \tag{25}$$

Here, with allowance for (21) and (22), we have

$$J_1 = \int_0^{\hat{t}_*} b^2\dot{b}^3\, d\hat{t} = Mh^2\Big[(t_0 + 5\mathrm{e}^{-t_0} - 5)M + \frac{1}{5}(\mathrm{e}^{5\ln b_*} - 1)\Big],$$

$$J_2 = \int_0^{\hat{t}_*} b\dot{b}^2\, d\hat{t} = M^2 h\Big(t_0 + 2\mathrm{e}^{-t_0} + \frac{1}{3}\mathrm{e}^{3\ln b_*} - \frac{11}{6} - \frac{1}{2}\mathrm{e}^{-2t_0}\Big).$$

The left side of Eq. (25) refers to the work done to expand the pores to the radius R_*, and the right side refers to the components of this work (disregarding the energy spent to form the free surface of the pore). Because the initial store of free elastic energy in the layer $1 < \bar{r} < a_* R_*^{-1} = b_*\beta_0^{-1}$ of the cell should be not smaller than $\Delta W(b_*)$ for the pore radius to reach the value of R_*, determining the value of $\Delta F = F(b) - F(b_*)$ from (13) and (17) for $b = 1$ and substituting it into the left side of (25), one can obtain the energy inequality

$$P_{i0}^2\Big(\frac{1}{3K_0}+\frac{1}{G}\Big)\Big[\frac{b_*^3}{\beta_0^3} - 1 + \frac{(1+\xi)^2}{2}\Big(1-\frac{\beta_0^3}{b_*^3}\Big)\Big] > 4\tau_*(b_*^3-1)\ln\beta_0^{-1} + 3\rho R_0^2[(1-\beta_0)b_*^3\dot{b}_*^2 - \beta_0 J_1] + 24\mu J_2 \tag{26}$$

that is the condition under which the pore grows to a given radius.

3. An analysis of (26) shows that, as $R_* \to R_0$ (i.e., $b_* \to 1$), the right side of the inequality tends to zero, and the left side to a certain finite value $[(3K_0)^{-1} + G^{-1}][\beta_0^{-3} + 0.5(1+\xi)^2(1-\beta_0^3)]P_{i0}^2$, whose physical meaning is as follows. According to (23), like ΔW [see (25)], the function ΔF contains, as a parameter, the initial radius of the pore R_0 [after substitution of ΔF into (25), R_0 is cancelled and it is absent in (26)]; therefore, as $R_* \to R_0$, the expression for $\Delta F(b_*)$ is reduced to the form

$$\Delta F_0 = \Delta F(R_0) = \frac{2}{3}\pi R_0^3\Big(\frac{1}{3K_0}+\frac{1}{G}\Big)\Big[\beta_0^{-3} - 1 + \frac{(1+\xi)^2}{2}(1-\beta_0^3)\Big]P_{i0}^2. \tag{27}$$

It follows from (27) that ΔF_0 is the elastic free energy necessary to do work to increase the pore from $R = 0$ to R_0. However, since at the initial moment (at the moment when an elastic-stress field forms in the cell), the pore radius is already equal to R_0 by definition, not only the elastic energy released from the spherical layer $4\pi(a_*^2 - a_0^2)/3$, but also the initial "store" of energy ΔF_0 will be spent for its expansion from the initial radius R_0 to R_*. Therefore, a certain growth of pores is possible until the initial "stock" of energy ΔF_0 is spent at small values of P_{i0} which do not satisfy the condition $F(R_0) - F(R_*) > W(\Delta R)$, where $\Delta R = R_* - R_0$.

Thus, relations (24) and (26) allow one to determine the degree of pore extension b_* and the relaxation time of the volume-averaged negative pressure T_1 of the sample with the use of given values of the negative pressure applied to the medium, its rheological parameters, the initial sizes of the pores, and their initial volume (or counting) concentration. In deriving relations (24) and (26), at the stage of plastic flow, the shear viscosity was assumed to have a certain constant value μ characteristic of the given medium. This assumption, which is based on analysis of experimental data, was used in [3, 9], where the growth of an isolated pore in the spherical layer of a viscoplastic medium, which expands as the pore grows, under the action of a constant negative pressure was considered. However, as was noted, in a more exact approximation, μ should depend on the dynamics of loading of the medium, its rheological properties, and the character and

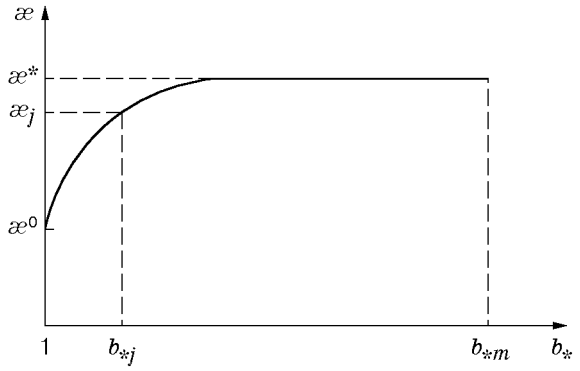

Fig. 4

amount of impurities. If this dependence is known in a particular case, one should substitute it into (13) instead of μ and perform the above procedures connected with the conditions for pore growth. In this case, the integration of Eq. (13) is complicated.

The results obtained are applicable to both liquid- and solid-plastic media. With allowance for the experimental data from [2, 9–11] on the physical parameters of media, the dependence of b_* and T_1 on $æ = P_{i0}/\tau_*$ was estimated by means of (24) and (26) for the following materials:

— gelatin jelly ($\tau_* = 10^4$ Pa, $\mu = 1$ Pa · sec, $R_0 = 10^{-4}$ cm, $n = 2000$ cm^{-3}, $\rho = 1.1$ g/cm^3, $K_0 = 4 \cdot 10^9$ Pa, and $G = 10^7$ Pa);

— lead ($\tau_* = 10^7$ Pa, $\mu = 40$ Pa · sec, $R_0 = 5 \cdot 10^{-5}$ cm, $n = 2000$ cm^{-3}, $\rho = 11.34$ g/cm^3, $K_0 = 4.44 \cdot 10^{10}$ Pa, and $G = 5.55 \cdot 10^9$ Pa);

— copper ($\tau_* = 6 \cdot 10^7$ Pa, $\mu = 30$ Pa · sec, $R_0 = 5 \cdot 10^{-5}$ cm, $n = 2000$ cm^{-3}, $\rho = 8.9$ g/cm^3, $K_0 = 11.46 \cdot 10^{10}$ Pa, and $G = 4.1 \cdot 10^{10}$ Pa).

The following dependences have been obtained:

— for gelatin jelly, $T_1 = 7.1 \cdot 10^{-4}$ sec and $b_* = 1.05$ at $\hat{t}_* = 6.8 \cdot 10^{-6}$ sec and $æ = 3.5$, $T_1 = 6.8 \cdot 10^{-4}$ sec and $b_* = 1.14$ at $\hat{t}_* = 1.6 \cdot 10^{-5}$ sec and $æ = 4$, $T_1 = 5.34 \cdot 10^{-4}$ sec and $b_* = 1.28$ at $\hat{t}_* = 2.4 \cdot 10^{-5}$ sec and $æ = 5$, and $T_1 = 7.9 \cdot 10^{-5}$ sec and $b_* = 1.73$ at $\hat{t}_* = 4.38 \cdot 10^{-5}$ sec and $æ = 6$; for $æ \geqslant 6.2$, the quantity b_* grows at least to values exceeding the value of $b_m = a_m(\beta_0 R_0)^{-1} = (2\beta_0 R_0 n^{1/3})^{-1} = \beta_0^{-1}[\pi/(6\alpha_0)]^{1/3}$ corresponding to reaching the cell boundary by the external radius of the viscoplastic layer;

— for lead, $T_1 = 2.3 \cdot 10^{-5}$ sec and $b_* = 1.053$ at $\hat{t}_* = 2 \cdot 10^{-7}$ sec and $æ = 5$, $T_1 = 1.06 \cdot 10^{-5}$ sec and $b_* = 1.26$ at $\hat{t}_* = 4.3 \cdot 10^{-7}$ sec and $æ = 10$, and $T_1 = 8.07 \cdot 10^{-6}$ sec and $b_* = 2.85$ at $\hat{t}_* = 1.5 \cdot 10^{-6}$ sec and $æ = 12.7$; $b_* \geqslant b_m$ for $æ > 13$;

— for copper, $T_1 = 2.41 \cdot 10^{-6}$ sec and $b_* = 1.014$ at $\hat{t}_* = 8.7 \cdot 10^{-9}$ sec and $æ = 5$, $T_1 = 1.15 \cdot 10^{-6}$ sec and $b_* = 1.2$ at $\hat{t}_* = 3.7 \cdot 10^{-8}$ sec and $æ = 10$, $T_1 = 8.14 \cdot 10^{-7}$ sec and $b_* = 3.5$ at $\hat{t}_* = 2.1 \cdot 10^{-7}$ sec and $æ = 13.5$; $b_* \geqslant b_m$ for $æ > 13.6$.

It follows from the results presented above that the qualitative dependence $b_*(æ)$ (Fig. 4) is the same for any condensed media possessing viscoelastic-plastic properties. Here $æ^0$ is the minimum value of $æ$ at which the growth of cavitating pores begins, b_{*j} is the maximum degree of pore extension for a given value of $æ_j$, and $æ^*$ is the threshold level $æ$ at which the pore radius grows to values equal or exceeding $R_m = R_0\beta_0^{-1}(6\alpha_0/\pi)^{-1/3}$.

Thus, for each viscoelastic-plastic body, there are three ranges of loading by negative pressure: $æ < æ^0$ (the pore sizes remain constant during the volume deformation of the body), $æ^0 < æ < æ^*$ (the deformation is accompanied by an insignificant extension of the pores: $\alpha\alpha_0^{-1} \leqslant 10$), and $æ > æ^*$ [the unrestricted growth of the pores to the sizes $R_m \geqslant R_0\beta_0^{-1}(6\alpha_0/\pi)^{-1/3}$]. It is noteworthy that a certain qualitative analogy exists in the growth of bubbles from cavitational nuclei in liquids under the action of negative pressure [12], where there are also three ranges of negative-pressure values corresponding to the stable state of the radius of a cavitational nucleus, its unlimited extension, and the unbounded growth of a cavitational bubble.

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